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Interface conditions for the observation of spin-wave resonance in bilayer exchange-coupled ferromagnetic films

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Abstract. It is well known that spin-wave resonance (SWR) phenomena can be observed in single-layer ferromagnetic films provided that the sample exhibits certain (surface or volume) inhomogeneities in its respective magnetic properties. Here, we show theoretically that the observation of SWR in bilayer (exchange-coupled) ferromagnetic films is possible even if both constituent sublayers are homogeneous provided that their bulk characteristics differ appropriately from each other.

1. Introduction

Multilayer magnetic films have long been of particular interest (see, e.g., the review in [1]), and in the last few years a real fascination with these structures has occurred. This is mainly due to the recent developments in materials science achieved by the application of sophisticated evaporation techniques to produce good-quality layered magnetic structures (multilayers and superlattices). Among the various properties of multilayer magnetic films studied in the literature, spin-wave excitations (see, e.g., [2–5]) and in particular their resonance investigations [6–10] have become of great importance.

In this work we shall study theoretically the simplest case of a multilayer film, i.e. the case of *two* ferromagnetic layers directly coupled (by exchange interactions) at their interface. We are mainly interested in the spectrum of standing spin waves (modes) supported by this film structure and the effects exerted thereon by the interface coupling. We shall show that in contrast to the case of a single-layer film, it is not necessary to have surface or interface inhomogeneities in order to observe a multipeak ferromagnetic resonance spectrum in the bilayer film. Even in the absence of such inhomogeneities one can expect to observe several resonance lines due to spin-wave modes if the magnetisations of both sublayers are mutually tilted. A handy criterion for establishing the experimental conditions under which the multipeak spin-wave resonance (SWR) can be observed is also proposed.

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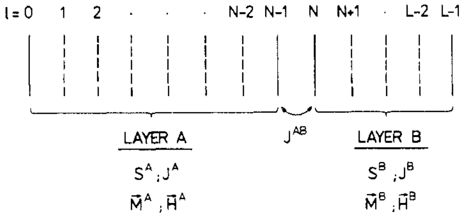

Figure 1. Model of a bilayer film consisting of two ferromagnetic layers A and B, coupled together by the interface exchange integral J^{AB}. N is the number of lattice planes in sublayer A, whereas $L-N$ is the number of lattice planes in sublayer B. The ferromagnetic sublayers are assumed to differ with respect to the following magnetic properties: their spin numbers S^A and S^B, respectively, nearest-neighbour exchange integrals J^A and J^B, magnetisations $\boldsymbol{M}_s^A$ and $\boldsymbol{M}_s^B$ and effective fields $\boldsymbol{H}^A$ and $\boldsymbol{H}^B$.

2. The model

Let us consider a sample (to be referred to henceforth as the *film*), consisting of two ferromagnetic materials in the shape of thin layers (sublayers A and B) each of homogeneous structure, extending in an unbounded manner in directions parallel to the surface of the film (fulfilling Born–Kármán periodic boundary conditions in these directions). In general, the two sublayers are assumed to have different magnetic properties; however, for simplicity, we shall assume that their crystallographic structures are identical. The two sublayers form one magnetic system owing to the exchange coupling assumed to exist at the interface separating them.

On the above assumptions, atoms lying in the same lattice plane parallel to the surface (to be termed in brief *a plane*) are in identical physical conditions, i.e. they are mutually equivalent, forming a magnetic sublattice. We shall treat *each* of the sublayers within the surface inhomogeneity approximation assuming that each sublayer consists of only two sublattices, namely one comprising its two surface planes and one comprising all its internal planes. The two sublayer surfaces which form the interface will be referred to as the *interface planes*. An atom is labelled by means of an index lj, where l is a number denoting the plane, and j is a two-dimensional vector lying in the plane of the film. As shown in figure 1, the index l takes the following values:

$l = 0$	surface plane A
$l = 1, 2, \ldots, N-2$	internal planes A
$l = N-1$	interface plane A
$l = N$	interface plane B
$l = N+1, N+2, \ldots, L-2$	internal planes B
$l = L-1$	surface plane B.

Let us assume, in a semi-classical approximation, that a spin present in a lattice site can be represented as

$$S_{lj} = \begin{cases} S^A \boldsymbol{\gamma}^A & \text{for sublayer A} \\ S^B \boldsymbol{\gamma}^B & \text{for sublayer B} \end{cases} \tag{2.1}$$

where S^A and S^B are the respective spin numbers (in units of $\hbar$), and $\boldsymbol{\gamma}^A$ and $\boldsymbol{\gamma}^B$ denote versors of the quantisation directions shared by all the spins of the sublayer A or B respectively. The directions $\boldsymbol{\gamma}^A$ and $\boldsymbol{\gamma}^B$ are to be determined by a minimisation procedure

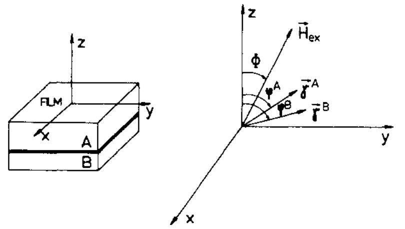

Figure 2. Choice of x, y, z coordinates. γ^A and γ^B are unit vectors of magnetisation directions in the sublayers A and B, respectively. H_{ex} is the external static magnetic field.

of the appropriate ground-state energy of the system as a whole; however, in a first approximation we may assume that they can be found separately by a standard minimisation procedure for each of the two sublayers independently. The externally applied static field H_{ex} can take any orientation with regard to the film surface (figure 2) although remaining within the y–z plane. The effective field for a given sublayer is defined as the sum of the external static field and the (sublayer) demagnetisation field:

$$H^{A,B} \equiv H_{ex} - \gamma_z 4\pi M_s^{A,B} \cos \varphi^{A,B}. \tag{2.2}$$

Here, to emphasise the interface effects, we neglect in our considerations both bulk and surface anisotropy fields; their eventual inclusion into the model poses no difficulty at all from the mathematical point of view (cf [11–15]). Consequently, the magnetisation directions are parallel to the intrinsic effective fields, equation (2.2), and remain within the y–z plane (see figure 2). Therefore the tilts φ^A and φ^B of the sublayer magnetisations can be found from the equations

$$H_{ex}/4\pi M_s^{A,B} = (\sin 2\varphi^{A,B})/2 \sin(\varphi^{A,B} - \Phi). \tag{2.3}$$

Above, $\varphi^{A,B}$ and Φ are the angles measured from the normal to the film to the sublayer magnetisation $M_s^{A,B}$ and the external field H_{ex}, respectively.

We perform our calculations within the framework of the Heisenberg localised spin model assuming exchange (nearest-neighbour) interaction and a Zeeman Hamiltonian in standard form:

$$\hat{H} = -\sum_{lj \neq l'j'} J_{ll'} \hat{S}_{lj} \cdot \hat{S}_{l'j'} - g\mu_B \sum_{lj} H_l^{eff} \cdot \hat{S}_{lj} \tag{2.4}$$

where summation extends over pairs of neighbouring spins; the exchange integral $J_{ll'}$ between nearest neighbours situated respectively in layers l and l' is assumed to be

$$J_{ll'} = \begin{cases} J^A & \text{if both interacting spins belong to the sublayer A} \\ J^{AB} & \text{if interacting spins belong to different sublayers (i.e. coupling through the interface)} \\ J^B & \text{if both interacting spins belong to the sublayer B} \end{cases} \tag{2.5}$$

and the effective field H_l^{eff} has the meaning of the field defined by equation (2.2):

$$H_l^{eff} = \begin{cases} H^A & \text{for } l \text{ belonging to sublayer A} \\ H^B & \text{for } l \text{ belonging to sublayer B.} \end{cases} \tag{2.6}$$

The Hamiltonian (2.4) is diagonalised here by applying the procedure described in detail in a separate paper [11]. The diagonalisation results in establishing the wavefunctions and spin-wave excitation energies permitted in our bilayer film. In the following section we restrict ourselves to presenting results concerning *standing* spin waves only, leaving the discussion of propagating spin waves for a separate paper.

3. Standing spin waves

For standing spin-wave modes one assumes the in-plane wavevector components to be zero: $k_x = k_y \equiv 0$; consequently, the amplitudes of the spin-wave modes are functions of the remaining non-zero wavevector component (perpendicular to the film surface) only. From the diagonalisation procedure it follows that the spin-wave mode amplitudes u_l have to satisfy the following set of homogeneous equations (cf [11, 12]):

$$\begin{aligned}
&(x-1)u_0 - u_1 = 0 && l=0\\
&-u_0 + xu_1 - u_2 = 0 && l=1\\
&\dots\\
&-u_{N-2} + (x-b)u_{N-1} - \rho u_N = 0 && l=N-1\\
&-\rho' u_{N-1} + (y-c)u_N - u_{N+1} = 0 && l=N\\
&-u_N + yu_{N+1} - u_{N+2} = 0 && l=N+1\\
&\dots\\
&-u_{L-3} + yu_{L-2} - u_{L-1} = 0 && l=L-2\\
&u_{L-2} + (y-1)u_{L-1} = 0 && l=L-1
\end{aligned} \qquad (3.1)$$

where we have introduced the following notation:

$$x = 2 + (g\mu_B \boldsymbol{H}^A \cdot \boldsymbol{\gamma}^A - E)/2S^AJ^Az \qquad y = 2 + (g\mu_B \boldsymbol{H}^B \cdot \boldsymbol{\gamma}^B - E)/2S^BJ^Bz \qquad (3.2)$$

$$b = 1 - (S^BJ^{AB}/S^AJ^A)\boldsymbol{\gamma}^A \cdot \boldsymbol{\gamma}^B \equiv 1 - (S^BJ^{AB}/S^AJ^A)\cos(\varphi^B - \varphi^A) \qquad (3.3)$$

$$c = 1 - (S^AJ^{AB}/S^BJ^B)\boldsymbol{\gamma}^A \cdot \boldsymbol{\gamma}^B \equiv 1 - (S^AJ^{AB}/S^BJ^B)\cos(\varphi^B - \varphi^A) \qquad (3.4)$$

$$\rho = \tfrac{1}{2}\sqrt{S^B/S^A}\,(J^{AB}/J^A)(1 + \boldsymbol{\gamma}^A \cdot \boldsymbol{\gamma}^B) \equiv \sqrt{S^B/S^A}\,(J^{AB}/J^A)\cos^2[\tfrac{1}{2}(\varphi^B - \varphi^A)] \qquad (3.5)$$

$$\rho' = \tfrac{1}{2}\sqrt{S^A/S^B}\,(J^{AB}/J^B)(1 + \boldsymbol{\gamma}^A \cdot \boldsymbol{\gamma}^B) \equiv \sqrt{S^A/S^B}\,(J^{AB}/J^B)\cos^2[\tfrac{1}{2}(\varphi^B - \varphi^A)]. \qquad (3.6)$$

The meaning of the quantities occurring in the above formulae is the following: z is the number of nearest neighbours situated in the adjacent plane, E is the energy of a given spin-wave mode, b and c denote the *interface-pinning* parameters, whereas ρ and ρ' are *effective interface-coupling* parameters. One sees that the pinning as well as the effective coupling interface parameters are functions of the *interface canting* angle $\varphi^B - \varphi^A$. This will turn out to have essential physical consequences.

To characterise the spin-wave modes of a bilayer film it is necesary to introduce two (perpendicular) wavevector components k_A and k_B, each of the two being assigned to one particular sublayer. They are to be defined by the following equalities (see [16]):

$$x \equiv 2\cos k_A \qquad y \equiv 2\cos k_B. \qquad (3.7)$$

From equations (3.2) and (3.7), the mode energy E can be expressed by either of the wavenumbers defined above:

$$E(k_A) = 4S^AJ^Az(1 - \cos k_A) + g\mu_B H^A \tag{3.8a}$$

or

$$E(k_B) = 4S^BJ^Bz(1 - \cos k_B) + g\mu_B H^B. \tag{3.8b}$$

Equating expressions (3.8*a*) and (3.8*b*) one obtains the following relation, to be satisfied between k_A and k_B:

$$1 - \cos k_B = (S^AJ^A/S^BJ^B)(1 - \cos k_A) + (g\mu_B/4S^BJ^Bz)(H^A - H^B). \tag{3.9}$$

This relation allows us to eliminate from our considerations either of the wavenumbers involved, if necessary.

The set of equations (3.1) has been solved strictly by one of us [16, 17] applying the recurrential interface rescaling method especially invented for that purpose. The functions u_l are contained in [17] explicitly as well as the characteristic equation quantising the mode numbers k_A, k_B. Here, we shall refrain from adducing the formulae for u_l and refer the reader to [17] since it is not our aim to proceed to a numerical analysis at this stage of investigation. On the contrary, we shall show that relevant qualitative, physically meaningful conclusions can be reached by analysing the properties of the set (3.1) without having recourse to the explicit form of u_l. The characteristic equation takes the following form:

$$\{\cos[\tfrac{1}{2}(2N+1)k_A]/\cos[\tfrac{1}{2}(2N-1)k_A] - b\} \times [\![\cos\{\tfrac{1}{2}[2(L-N)+1]k_B\}/\cos\{\tfrac{1}{2}[2(L-N)-1]k_B\} - c]\!] = \rho\rho'. \tag{3.10}$$

As one notes, the set of permitted values of k_A (and, by equation (3.9), the set of k_B equivalent thereto) is determined by all four interface parameters, equations (3.3)–(3.6), and by the thicknesses of the sublayers and the film.

4. Spin-wave resonance

It will be remembered (see, e.g., [12, 13, 18]) that the intensity of the resonance line corresponding to a given mode is directly proportional to the squared sum over its amplitudes across the film, i.e.

$$I \sim \left| \sum_{l=0}^{L-1} u_l \right|^2. \tag{4.1}$$

On summation of all the equations in the set (3.1) we obtain

$$(x + y - 4)\sum_{l=0}^{L-1} u_l + (x - y)\left[\sum_{l=0}^{N-1} u_l - \sum_{l=N}^{L-1} u_l\right] = (b + \rho' - 1)u_{N-1} + (c + \rho - 1)u_N. \tag{4.2}$$

Equation (4.2) allows 'extraction' of the sum occurring in equation (4.1) and thus enables us to draw certain essential conclusions relating to the conditions for the observation of SWR. However, prior to analysing equation (4.2), we shall make reference to a basic property of the SWR phenomenon in a single-layer film; we have in mind the fact that the whole observable SWR spectrum (the set of lines with sufficiently great intensities) lies at the bottom of the magnon band, i.e. within the energy region where the so-called long-wavelength approximation ($k \simeq 0$) is sufficiently good for the interpretation of the SWR spectrum. Hence it is only logical to assume that, in the case of bilayer films, *two* energy regions exist where absorption of the microwave field can be expected to be easily perceptible, namely those regions that correspond to $k_A \simeq 0$ and $k_B \simeq 0$. Thus, before proceeding any further, it is essential to ascertain whether the magnon bands of the component materials A and B constituting the bilayer system overlap or are well separated; in the former case (overlap) the two *resonance regions* will also overlap, meaning that the two sublayers will participate simultaneously in resonant absorption, whereas in the latter case (no overlap, and two mutually remote regions of resonance) either sublayer will come into resonance individually. Moreover, regarding equation (4.2) we have to keep in mind that, if the sum $\Sigma_l u_l$, performed within the film or within one of the two sublayers, vanishes for a given mode, the mode in question cannot interact with the alternating magnetic field and, consequently, no absorption takes place in the respective system or subsystem.

Let us first consider the case when the magnon bands of the two component materials (A and B) overlap almost completely, i.e. when $x - y \simeq 0$; the second term on the left-hand side of equation (4.2) is then negligible and therefore the right-hand side expresses directly the resonance intensity. Obviously, this intensity is composed of two terms, originating respectively in the sublayer A (the term of amplitude u_{N-1}) and in the sublayer B (the term in u_N). Note that in this case the structure of (4.2) becomes identical with the structure obtained for a single-layer film (see [12], equation (6.7)) within the surface inhomogeneity model, the only difference residing in the fact that now the surface amplitudes are replaced by the respective interface amplitudes u_{N-1} and u_N. From this analogy we conclude that the condition sufficient for SWR observation is that the coefficients of the functions u_{N-1} and u_N must be *non-zero*. With regard to equations (3.3)–(3.6), these coefficients are, respectively,

$$b + \rho' - 1 = \tfrac{1}{2}\sqrt{S^A/S^B}\,(J^{AB}/J^B)\{1 + [1 - 2(S^B/S^A)^{3/2}(J^B/J^A)]\cos\delta\} \tag{4.3a}$$

$$c + \rho - 1 = \tfrac{1}{2}\sqrt{S^B/S^A}\,(J^{AB}/J^A)\{1 + [1 - 2(S^A/S^B)^{3/2}(J^A/J^B)]\cos\delta\} \tag{4.3b}$$

where $\delta \equiv \varphi^B - \varphi^A$ denotes the *interface canting angle* of sublayer magnetisations. Equations (4.3) show clearly that the resonance intensities are dependent not only on the interface characteristics J^{AB} and δ but moreover on the bulk properties of both sublayers (the ratios J^A/J^B and S^A/S^B) and that 'critical' resonance—the situation when one line only is excited, $k_A \simeq k_B = 0$ (requiring (4.3*a*) and (4.3*b*) to vanish *simultaneously*)—occurs when the following two conditions are fulfilled:

$$(J^B/J^A)^2 = (S^A/S^B)^3 \qquad \delta = 0. \tag{4.4}$$

We thus arrive at the conclusion that the observation of multipeak SWR is possible in this case provided that even one of the conditions (4.4) is unfulfilled.

We now proceed to the case when the magnon bands of the two materials A and B are markedly remote from each other, i.e. when we have quantitatively $|x - y| \gg 0$. Here, it is no longer possible to neglect the second term on the left-hand side of (4.2) as a whole; nonetheless, we are always justified in neglecting *one* of the sums $\Sigma_{l=0}^{N-1} u_l$ *or* $\Sigma_{l=N}^{L-1} u_l$ depending on the resonance region considered ($k_B \simeq 0$ or $k_A \simeq 0$, respectively). The remaining term, combined with the first term, again leads to the conclusion that the resonance line intensities are determined by the right-hand side of

(4.2). However, on the assumption made, one of the two terms occurring there can be neglected while the other term expresses the absorption taking place in the respective sublayer. As a further result, we can expect the presence of two critical resonances, when *either* (4.3*a*) *or* (4.3*b*) equals zero. These resonances will correspond to usual ferromagnetic resonance lines from sublayer A or sublayer B. For (4.3*a*) to vanish the condition to be fulfilled reads

$$(\cos\delta)_A = [2(J^B/J^A)(S^B/S^A)^{3/2} - 1]^{-1} \tag{4.5a}$$

whereas that for (4.3*b*) to vanish is

$$(\cos\delta)_B = [2(J^A/J^B)(S^A/S^B)^{3/2} - 1]^{-1}. \tag{4.5b}$$

If either of these conditions is unfulfilled, multipeak SWR absorption will take place in the respective sublayer. However, we still have to envisage the situation when the two conditions (4.5*a*) and (4.5*b*) are *unfulfilled simultaneously*; the total resonance spectrum will then consist of SWR spectra of the *two* sublayers.

Above, we have determined the conditions for the occurrence of critical resonances. Our chief aim in doing so was to show that a departure from these conditions constitutes the sufficient condition for SWR—our principal concern in this subsection. It should be noted that none of the conditions (4.4), (4.5*a*) and (4.5*b*) involves the interface exchange integral J^{AB}, meaning that the strength of interface exchange coupling has no influence on the occurrence or non-occurrence of SWR; it is only necessary that $J^{AB} \neq 0$, in order that equations (4.3) do not have to vanish identically. Thus, we have shown indeed that differences in the individual *bulk* characteristics of the two sublayers suffice for SWR to arise even in the absence of inhomogeneities of the type of surface or interface pinning. This was the purpose of our work: to show that SWR in bilayer magnetic films can originate in other sources, qualitatively distinct from the *classical* ones (from the surface- and interface-pinning anisotropies which were not discussed in this paper). In a separate paper, we shall present various numerically calculated SWR spectra obtained for several sets of the relevant parameters of the theory.

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